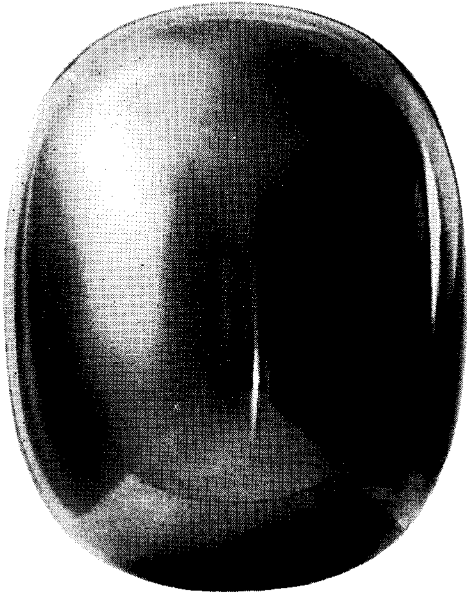
THE SUPER-EGG™



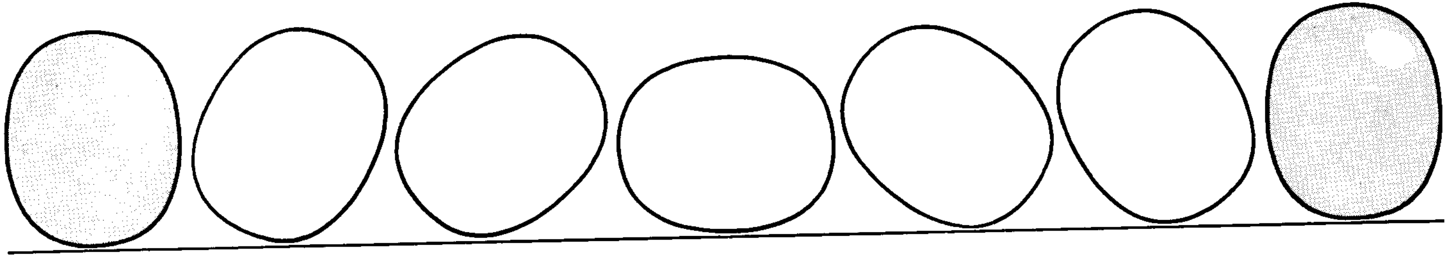
Invented by
PIET HEIN
DENMARK

ENGLISH – Available in many languages

THE SUPER-EGG is a sculpture, a plaything, and an amulet.

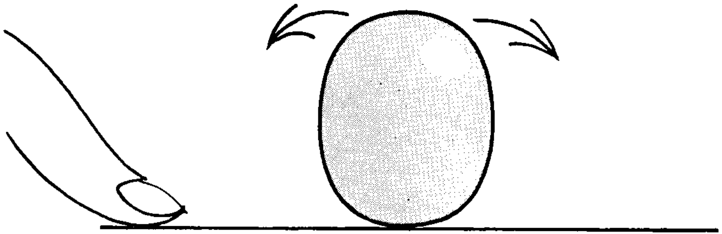
It is a spatial form of the super-ellipse, which is an exact curve related to both the circle and the square.

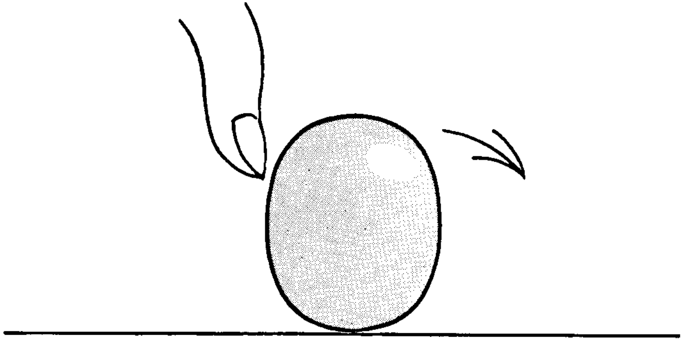
From the super-ellipse the super-egg derives its simple sculptural shape and the remarkable faculty of being able to stand upright on either end — without cheating, as in the case of Columbus' egg. This makes it a versatile and nerve-soothing plaything.



Stand the super-egg upright on a stiff foundation, for instance this booklet.
Move the foundation in such a way that the super-egg turns half a somersault and ends standing top-end down.

It is difficult at first, but as soon as you have managed this trick once it is surprisingly easy and you can go on to devise an endless variety of tricks.





When you have mastered the first trick, you will be acquainted with the super-egg and can proceed to make it turn a whole somersault, one-and-a-half somersaults, and so on.

There are four techniques for getting the super-egg started, and each of these gives different tricks: 1. move the foundation, 2. push the super-egg with your finger, 3. tilt one end of the foundation, 4. press the top of the upright super-egg downwards with your finger tip so that the lower end of the egg moves away from you, then it will rotate backwards, turning half a somersault and so on, to end standing on the other end or the same one.

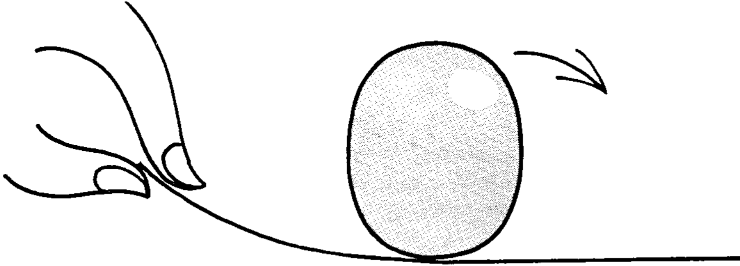
There are an infinite variety of tricks. You can move the foundation so that the super-egg falls on its side and rises again in the same position—or turns a pre-set number of somersaults backwards and forwards and ends standing upright. You can begin with the super-egg lying flat and shake the foundation so as to make it stand up. You can roll the super-egg up a slope so that it rolls back and resumes its upright position.

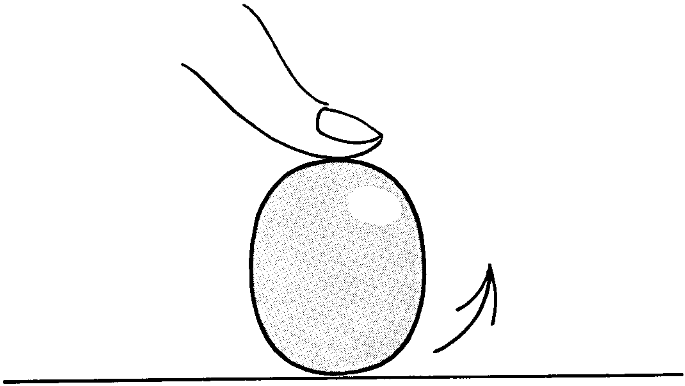
You can do the tricks with two or more super-eggs at a time.

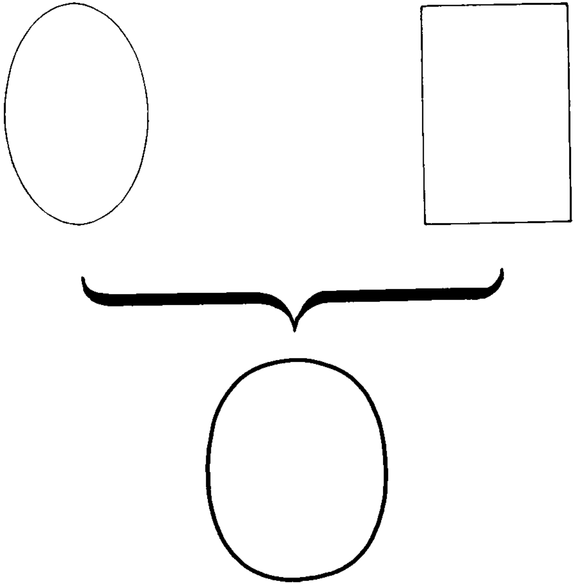
With two super-eggs you can perform special tricks—for instance, let one of them roll and topple over the other and then end standing up, or let both of them end standing up.

Competitions should concentrate on a set trick, players competing to carry out the set trick first, or competing to carry it out the greatest number of times in succession.

Produced in Denmark
by SKJØDE of SKJERN
exclusively for
PARKER BROTHERS, INC.,
SALEM, MASS., USA







Condensed from

SCIENTIFIC AMERICAN

THE SUPER-ELLIPSE

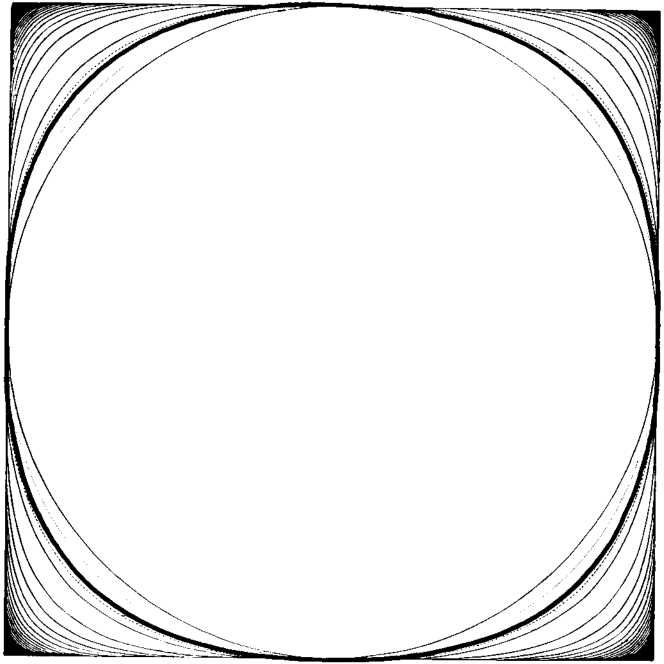
Civilized man is surrounded on all sides, indoors and out, by a subtle, seldom-noticed conflict between two ancient ways of shaping things: the orthogonal and the round. Cars on circular wheels, guided by hands on circular steering wheels, move along streets that intersect like the lines of a rectangular lattice. Buildings and houses are made up mostly of right angles, relieved occasionally by circular domes and windows. At rectangular or circular tables, with rectangular napkins on our laps, we eat from circular plates and drink from glasses with circular cross sections. We light cylindrical cigarettes with matches torn from rectangular packs, and we pay the rectangular bill with rectangular bank notes and circular coins.

The two extremes, the rectangular and the round, have been in conflict throughout the pattern of civilization. Why do we always choose one of the extremes? Because we prefer simple shapes! But there *is* a simple shape between the ellipse and the rectangle, related to them both. It is the super-ellipse.

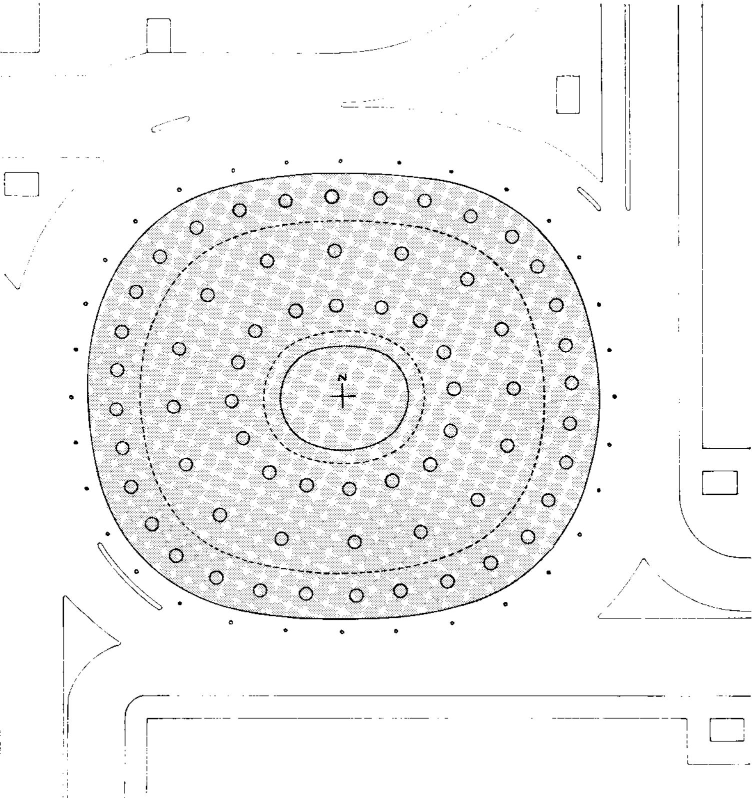
Even our games combine the orthogonal and the round. Most outdoor sports are played with spherical balls on rectangular fields. Indoor games, from pool to checkers, are similar combinations of the round and the rectangular. Wherever one looks the scene swarms with squares and circles and their affinely stretched forms: rectangles and ellipses.

The Danish writer and inventor Piet Hein recently asked himself a fascinating question: What is the simplest and most pleasing closed curve that mediates fairly between these two clashing tendencies? Originally a scientist, Piet Hein (he is always spoken of by both names) is well known throughout Scandinavia for his 24 enormously popular volumes of gracefully aphoristic poems and for his writings on scientific and humanistic topics. Norbert Wiener's last book, "God and Golem, Inc.", is dedicated to him.

The question he asked himself had been suggested by a knotty city-planning problem that first arose in 1959 in Sweden. Many years earlier Stockholm had decided to raze and rebuild a congested section of old houses and



Super-ellipses are the simplest possible curves between the ellipse and the rectangle. They form a transition between the two, combining the character of both. The illustration shows their relationship when height and width are equal: the square and the super-circle.



narrow streets in the heart of the city. After 1945 this enormous and costly program got under way. Two broad new traffic arteries running north-south and east-west have been cut through the center of the city. At the intersection of these avenues a large rectangular space about 200 yards long is being laid out. At its center will be an oval basin with a fountain surrounded by a large oval pool containing several hundred smaller fountains. Daylight will filter through the pool's translucent bottom into an oval self-service restaurant, below street level, surrounded by oval rings of pillars and shops. Below that there will be two more oval floors for dining and dancing, cloakrooms and kitchen.

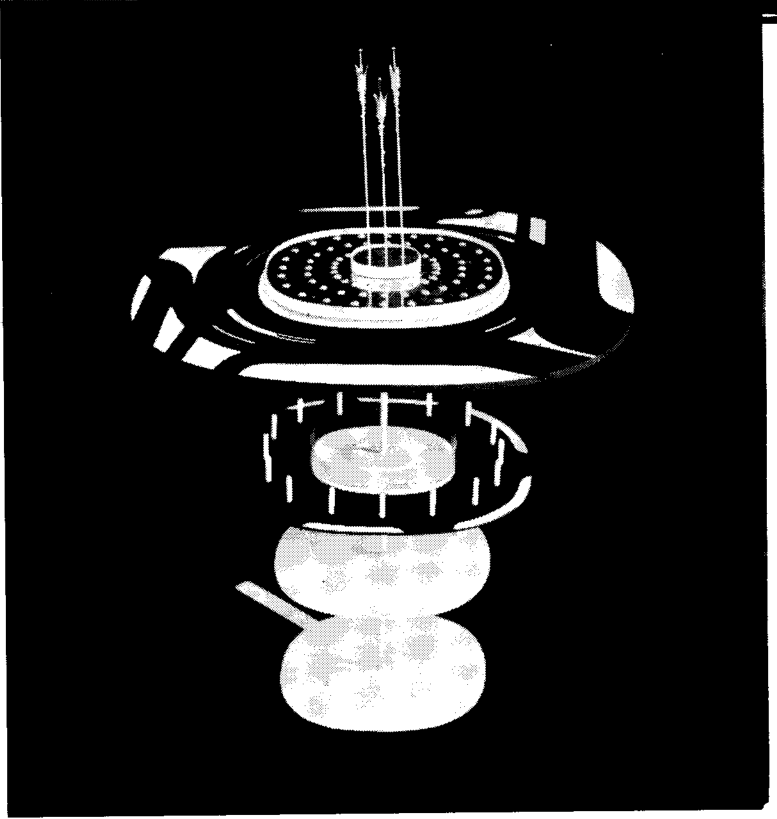
In planning the exact shape of this center the Swedish architects ran into unexpected snags. The ellipse had to be rejected because its pointed ends would interfere with smooth traffic flow around it; moreover, it did not fit harmoniously into the rectangular space. The city planners next tried a curve made up of eight circular arcs, but it had a patched-together look with ugly "jumps" of curvature in eight places. In addition, plans called for nesting different

The super-ellipse set in the rectangular heart of the city.

sizes of the oval shape, and the eight-arc curve refused to nest in a pleasing way.

At this stage the architectural team in charge of the project consulted Piet Hein. It was just the kind of problem that appealed to his combined mathematical and artistic imagination, his sense of humor and his knack of thinking creatively in unexpected directions. What kind of curve, less pointed than the ellipse, could he discover that would nest pleasingly and fit harmoniously into the rectangular open space at the heart of Stockholm?

Piet Hein's novel answer is a family of curves of which the ellipse (and the circle) and the rectangle (and the square) are special cases. These super-ellipses, as he called these new curves, vary between these two extremes which are reached when the exponent in their general formula becomes 2 and ∞ respectively. For the Stockholm super-ellipse Piet Hein settled on the exponent $2\frac{1}{2}$. With the help of a computer, 400 coordinate pairs were calculated to 15 decimal places, and larger, precise curves were drawn in many different sizes, all with the same height-width ratios



The super-ellipse building, Stockholm's new centre (principle model).
On the back of the cover: a couple of the super-ellipses carried out in concrete.

(to conform with the proportions of the open space at the center of Stockholm). The curves proved to be strangely satisfying, neither too rounded nor too orthogonal, a happy blend of elliptical and rectangular beauty. Moreover, such curves could be nested, to give a strong feeling of harmony and parallelism between the concentric ovals. Piet Hein calls all such curves with exponents above 2 "super-ellipses". Stockholm immediately accepted the $2\frac{1}{2}$-exponent super-ellipse as the basic motif of its new center. The huge, partly subterranean structure is now under construction. When the entire center is finally completed–perhaps in 1970–it is expected to be one of the great tourist attractions of Sweden.

Meanwhile Piet Hein's super-ellipse has been enthusiastically adopted by industries in Denmark, Sweden, Norway and Finland that turned to Piet Hein for solutions to various orthogonal-v.-circular problems, and in recent months he has been working on super-elliptical furniture, dishes, coasters, lamps, silverware, textile patterns etc.

Incidentally, one must not confuse the Piet Hein

Model city of super-elliptic buildings.
More open than a similar city composed of rectangular structures.

Printed in Denmark by Strandbygaard

super-ellipse with the superficially similar potato-shaped curves one often sees, particularly on the face of television sets. These are seldom more than oval patchworks of different kinds of arc, and they lack any simple formula that gives aesthetic unity to the curve.

A solid model of a prolate spheroid will no more balance upright on either end than a chicken egg will, unless one applies to the egg a stratagem usually credited to Columbus.

What does this have to do with super-eggs?

Well, Piet Hein discovered, that a super-egg, balances immediately on either end without any sort of skulduggery!

Consider the super-egg. It looks as if it should topple over, but it does not. This spooky stability of the super-egg (on both ends) can be taken as symbolic of the super-elliptical balance between the orthogonal and the round.

Martin Gardner

Super-elliptic table - and games with super-eggs.

Printed in Denmark by Strandbygaard

WHY CAN THE SUPER-EGG STAND ON END?

What are the conditions for an object to be able to keep its balance?

Have you ever tried to use a rocking chair to stand on while putting up a curtain rod?

Even if you haven't, you'll know that the rocking chair will lose its balance and throw you off.

Why?

Because the centre of gravity is too high in relation to the centre of the rockers.

If the centre of gravity of rocking chair plus body is lower than the centre of the rockers, the whole will be in balance: if it is higher, it is out of balance. Simply because small side movements will elevate the centre of gravity in the former case and lower it in the latter.

In an ordinary egg, the rockers are the surface of the shell at the end. It curves like a small segment of a sphere having a shorter radius than the sphere which is as large as the egg lengthwise. Therefore it has its centre of curvature below the centre of gravity of the egg. This means that the egg cannot stand on end.

But as soon as the cross section of the egg, the ellipse, becomes a super-ellipse (and the egg, the ellipsoid, becomes a super-ellipsoid), the radius of curvature at the end becomes greater than the distance to the centre of gravity, and the egg can keep its balance—just like a rocking chair, where the centre of the rockers lies above the centre of gravity.

The radius of curvature becomes greater! This is putting it mildly. One of the many peculiar properties of the super-ellipse is that the radius of curvature at the ends is infinitely large. The curve is not straight over any finite section, but right at the end it is! *)

This means that any super-egg, no matter how little it differs from the ellipsoid, the chicken egg, and regardless of its height in proportion to its width, can balance on either end—indeed that any number of such eggs can theoretically balance on top of one another.

---

*) The super-ellipse's formula is:

$$\left|\frac{x}{a}\right|^p + \left|\frac{y}{b}\right|^p = 1$$

The super-egg's formula is:

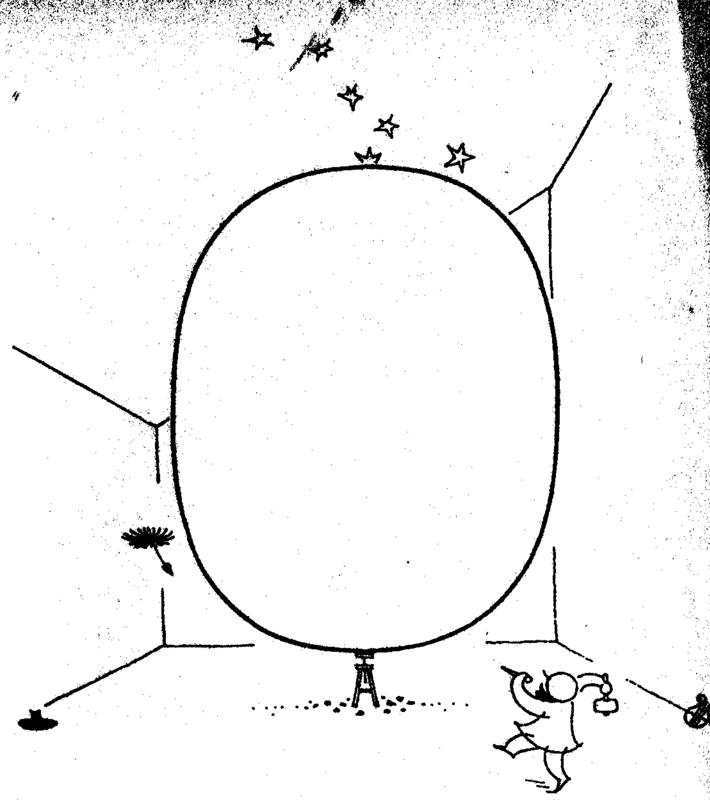
$$\left|\frac{\sqrt{x^2+y^2}}{a}\right|^p + \left|\frac{z}{b}\right|^p = 1$$

where p is over 2.

When $p = 2$, these are the formulae for the ordinary ellipse and the ordinary ellipsoid, the chicken egg. In these the radius of curvature at the pointed ends is shorter than the distance to the centre of gravity. That is the reason why the chicken egg cannot balance on end.

But as soon as p becomes the slightest bit greater than 2—for instance, 2,000000001 the strange thing occurs that the radii of curvature at the pointed ends immediately become infinite.

The Super-Egg World Center, Skjern, Denmark



There is
one art,
no more,
no less:
to do
all things
with art-
lessness.

Piet Hein

The Super-Egg World Center, Skjern, Denmark